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Power measuring in Dichotomous multi-type games

We consider multi-type games, which are a generalization of the classical cooperative game. This framework models games for which the following conditions are met: (i) Several, say r non-ordered types (not levels) of support are allowed in the input. Each player chooses one of the r types of support. The players' choices then lead to a choice configuration. (ii) The characteristic function maps each choice configuration to its value. By their nature, classical cooperative games allow two levels of inputs. For example, in a transferable utilities game, each player chooses either "to participate" or "not", and a characteristic function maps each choice configuration to a real value. In a simple voting game, when a single alternative, such as a bill or an amendment, is pitted against the status quo, the alternative is approved only on the basis of the votes cast by those who are in favor. In other words, voting "yes" and "no" are the only feasible alternatives. It is well known that a number of interesting questions of economics, politics and more generally social sciences cannot be described by a classical cooperative game. Ternary voting games or $(3; 2)$ games that allow for a distinct third option have been directly extended to (j, k) games by Freixas and Zwicker (2003). In this setting, each player expresses one of j ordered possible levels of input support, and the out-put consists not of a real value but of one of k possible levels of collective support. Another model of game with ordered inputs is the so-called multichoice game introduced by Hsiao and Raghavan (1993), in which each player is allowed to have a given number of effort levels, each of which is assigned a nonnegative weight. The weight assigned to an effort level leads to an ordering on the set of effort levels. Any choice configuration is then associated with a real value. An example of a model in which alternatives in the inputs are not totally ordered is the one formalized by Laruelle and Valenciano (2012) in which the four possible alternatives are "yes", "no", "abstention" and "non-participation". The collective decision is dichotomous, *i.e.*, either the proposal is accepted or rejected. This model and the other models above are particular cases of the more general framework of games on lattices developed by Grabisch and Lange (2007). A model of a game that does not fit among the class of games on lattices is that developed by Bolger (1986), called games with n players and r alternatives, or simply r -games. In such games, there are r possible input alternatives that are not ordered. Each alternative j attracts its own coalition of supporting voters. A configuration, which is a partition of the set of players into r subsets, is then associated with an r -tuple of cardinal values. The component j represents the value of the coalition of the configuration that has chosen the input j . This model is related to ours in the sense that the set of inputs is not ordered. No alternative is a priori more favorable than another. However, the models differ in their outputs. Our output consists of a single value.

. As usual, a central concern in game theory is to define a value or solution concept for a game. The prominent value is the well-known Shapley value. In the particular context of simple games, different theories of power have been proposed. The most famous are those of the Shapley-Shubik (1954) and the Banzhaf-Coleman (Banzhaf (1965) and Coleman (1971)) indices. These indices have been extended to the context of multiple alternatives in various games. They were defined for ternary voting games by Felsenthal and Machover (1997). For (j, k) games the extensions are due to Freixas (2005a) and Freixas (2005b). Our main concern is to extend and fully characterize the most commonly used voting power indices, including Shapley-Shubik, Banzhaf-Coleman, Deegan-Packel and others, when "dichotomous" multi-type games (*DMG*) are considered. *DMG* are particular cases of multi-type games where the output is dichotomous.