Complex magnetic ordering and associated topological Hall effect in two-dimensional metallic chiral magnets

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Study of metallic chiral magnets

- Materials where local moments interact via exchange interaction and Dzyaloshinskii-Moriya (DM) interaction.
- Competition between these interactions stabilizes a magnetic skyrmion phase.
- A periodic array of skyrmions (size $O(1) \text{nm}$) gives rise to an emergent magnetic field $O(10^3) \text{T}$. Several applications (efficient storage, transport) have been predicted.
- To study the magnetic and associated transport behaviour in 2D chiral magnet
  - Microscopic model (on a square lattice):
    \[
    \mathcal{H}_c = -J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - D \sum_{\langle i,j \rangle} (\hat{z} \times \hat{r}_{ij}) \cdot (\mathbf{S}_i \times \mathbf{S}_j) - B \sum_i S_i^z
    \]
  - Method
    - Simulated annealing with Monte Carlo based on Metropolis update.
    - Verify the low temperature MC results with variational calculation.
  - Results
    - Magnetic Properties: Spin texture, magnetisation, long-range spin correlation and spin chirality
    - Electronic Properties: Band structure, topological Hall effect
Magnetic properties: Ground state

- Spin configuration and structure factor
  - Three distinct phases: spiral, skyrmion crystal (SkX), and polarised phase.
  - $S_{xy}(\mathbf{Q})$ shows characteristic peaks of each phase.

- Energy, magnetisation, spin chirality
  - Our MC simulation correctly captures the ground state.
  - Magnetisation jumps across the phase transition points.
  - The SkX phase has finite spin-chirality.
Magnetic properties: Finite temperature

- Evolution of structure factor peak
  - The weight reduces with increasing temperature.
  - $T_c$ is estimated from the point of inflection.

- Behaviour of spin chirality
  - Thermal transition and crossovers between different phase.

<table>
<thead>
<tr>
<th></th>
<th>Spiral</th>
<th>SkX</th>
<th>SkL</th>
<th>FM</th>
<th>PM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S(Q)$</td>
<td>long range</td>
<td>long range</td>
<td>short range</td>
<td>long range</td>
<td>Zero</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Zero</td>
<td>Finite</td>
<td>Finite</td>
<td>Zero</td>
<td>Zero</td>
</tr>
<tr>
<td>$\sigma_{xy}$</td>
<td>Zero</td>
<td>Finite</td>
<td>Finite</td>
<td>Zero</td>
<td>Zero</td>
</tr>
</tbody>
</table>
Electronic properties: Band structure

\[ \hat{H}_e = - \sum_{\langle i,j \rangle, \sigma} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} - J_K \sum_i S_i \cdot s_i - \mu \sum_i n_i \]

In the limit \( J_K \to \infty \),

\[ \hat{H}_{\text{eff}} = - \sum_{\langle i,j \rangle, \sigma} t_{ij}^{\text{eff}} (d^\dagger_i d_j + \text{H.c.}) \]

where,

\[ t_{ij}^{\text{eff}} = te^{i a_{ij}} \cos \frac{\theta_{ij}}{2} \]

\[ a_{ij} = \arctan \frac{-\sin(\phi_i - \phi_j)}{\cos(\phi_i - \phi_j) + \cot \frac{\theta_i}{2} \cot \frac{\theta_j}{2}} \]

- For skyrmion unit cell \( \lambda \times \lambda \), there are \( 2\lambda^2 \) bands.
- The lower bands are gapped with Chern number \( C = -1 \).
- The uppermost two bands, (close to the van Hove singularity), touch each other and have \( C = \lambda^2 / 2 - 2 \).
\[ \hat{H}_e = - \sum_{\langle i, j \rangle, \sigma} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} - J_K \sum_i S_i \cdot s_i - \mu \sum_i n_i \]

\[ \sigma_{xy} = \frac{ie^2 \hbar}{N} \sum_\sigma \sum_{m,n \neq m} (f_m - f_n) \langle m | v_x | n \rangle \langle n | v_y | m \rangle \frac{1}{(E_m - E_n)^2 + \eta^2} \]

where,

\[ v_\mu = \frac{i}{\hbar} \sum_{j, \sigma} (t_{j,j+\mu} c^\dagger_{j,\sigma} c_{j+\mu,\sigma} - \text{H.c.}), \quad \mu = x, y \]

- Only the skyrmion phase exhibits THE.
- \( \sigma_{xy}^{\text{THE}} \) changes sign across van Hove singularity.
- \( \sigma_{xy}^{\text{THE}} \) shows quantised behaviour when \( \mu \) lies in the band gap.
- Effect of \( J_K \) and temperature are significant.
Transport behaviour: charge and spin topological Hall conductivities

\[ \sigma_{xy}^S = \frac{ie}{4\pi N} \sum_{\sigma} \sum_{m,n \neq m} (f_m - f_n) \left\langle m | J_x | n \right\rangle \left\langle n | v_y | m \right\rangle \frac{\langle m | J_x | n \rangle \langle n | v_y | m \rangle}{(\varepsilon_m - \varepsilon_n)^2 + \eta^2} \]

where,

\[ J_x = \frac{1}{2} \{ v_x, \text{diag}(S_1 \cdot \sigma, \ldots, S_N \cdot \sigma) \} \]

- \( \sigma_{xy} \) is symmetric for positive and negative values of \( \mu \). However, this is not the case for \( \sigma_{xy}^S \).
- For \( J_K \gg t \), \( \sigma_{xy} \) and \( \sigma_{xy}^S \) follow one another closely since only one species of electrons contribute to the Hall conductivities.
- For \( J_K \sim t \), \( \sigma_{xy} \) and \( \sigma_{xy}^S \) show different signatures as the electronic states in this limit exhibit overlapping behaviour.
Key message

- Skyrmions in chiral metallic magnets show unique transport signatures.
- Identifying skyrmion hosting materials based on their transport behaviour would be useful and may overcome the challenges of currently used complex imaging techniques.

THANK YOU

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