TOPOLOGICAL MAGNONS IN FLUX-STATE OF SHAstry-SUTHERLAND LATTICE

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Previous Studies and motivation

Spin Model on Shastry-Sutherland lattice
- Material Realization

Method: Holstein-Primakoff transformation

Magnon bands
- Flux state and Magnon band
- Canted Flux state and magnon band

Topological phase diagram
- Thermal Hall Conductivity and it’s derivative
- Temperature dependence of derivative in thermal Hall conductivity

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Previous Studies and motivation

- Umbrella phase in triangular lattice shows a lots of band topological phases \((\text{Phys. Rev. B 100, 064412})\)
- Frustrated lattices are ideal for realization of the non-coplanar magnetic structure.
- Non-coplanar magnetic ordering generally have a rich magnon band topological phases
- Shastry-Sutherland lattice is also a frustrated lattice and non-coplanar ordering is easy to achieve.

\[(\text{Phys. Rev. B 101, 214403})\]
Material Realization

- Low temperature spin-physics of Rare Earth Tetraborides can be modeled $J \approx J'$ Shastry-Sutherland lattice with

- In plane DM-interaction can be introduced using proximity effect, with a particular lattice symmetry.

- The perpendicular DM-interaction can be introduced by circularly polarized light.

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Spin Model on Shastry Sutherland lattice

\[ \mathcal{H} = J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J' \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + D \sum_{\langle i,j \rangle} (\mathbf{S}_i \times \mathbf{S}_j) + D' \sum_{\langle i,j \rangle} (\mathbf{S}_i \times \mathbf{S}_j) - B \sum_i S^z_i, \]
Method: Holstein Primakoff Transformation

- Step 1: Find the classical ground state

- Step 2: Rotate the coordinates at each lattice site, such that the z-axis aligned along the classical spin at that site.

- Step 3: Apply Holstein Primakoff transformation

\[
\hat{S}_{i,a}^+ = \sqrt{2S} \hat{a}_i,
\]
\[
\hat{S}_{i,a}^- = \sqrt{2S} \hat{a}_i^\dagger,
\]
\[
\hat{S}_{i,a}^z = S - \hat{a}_i^\dagger \hat{a}_i.
\]
Magnon Band in flux state

• Large perpendicular DM-interaction induces a Flux state.

• For Square lattice, Antiferromagnetic Heisenberg model, transition from Neel state to Flux state happens at,

\[ D_\perp = J' \]

• For Shastry Sutherland lattice if \( J \approx J' \), due to frustration, transition occurs at,

\[ D_\perp = 0.6J \]

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Magnon Band in flux state

• The four fold degeneracy along MX-line is due to Kramer’s degeneracy
  \[ \hat{m}_2' = \left\{ \tilde{M}_2 e^{i\pi \hat{S}_y} \tau | \delta_2 \right\} \]
  \[ (\hat{m}_2')^2 = e^{ik_x} = -1 \]

• The band at M-point is four fold degenerate due to, presence of symmetry,
  \[ \hat{m}_y = \left\{ \tilde{M}_y \tau | 0 \right\} \]

Which, maps one Kramer’s pair to another Kramer’s pair at M-point.

• The Goldstone modes are present due to the spontaneous breaking of continuous symmetry.

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Magnon Band in flux state

- At finite magnetic field, the four-fold degeneracy at M-point reduces to a two-fold degeneracy.
- The non-abelian Berry-curvature is non-zero.
- But the Chern number is zero.
Canted Flux State Magnon Bands in Canted

• In presence of in plane DM-interaction the classical ground state become Canted Flux state.

• Without any magnetic field there are still symmetry protected degeneracy in magnon band.

\[ J = 1.0, \, J' = 1.1, \, D_\perp = 0.8, \, D = 0.2, \, D_{||,s} = 0.05, \, D_{||,ns} = 0.1, \, B_z = 0.0. \]
Magnon Bands in Canted Flux State

- Magnetic field breaks all the symmetries and all bands are gapped,

\[ J = 1.0, \quad J' = 1.1, \quad D_\perp = 0.8, \quad D = 0.2, \quad D_{||,s} = 0.05, \quad D_{||,ns} = 0.1, \quad B_z = 0.3. \]
Magnon Bands in Canted Flux State

• Chern number,

\[ C_n = \frac{1}{2\pi} \int_{\text{BZ}} \Omega_{xy}^n(k) dk_x dk_y, \]

• Berry Curvature,

\[ \Omega_{xy}^n(k) = i\epsilon_{\mu\nu} \left[ \sigma_3 \frac{\partial T^\dagger_k}{\partial k_\mu} \sigma_3 \frac{\partial T_k}{\partial k_\nu} \right]_{nn} \]  

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Topological phase diagram

- Zoo of topological phases found, because the Berry phase of magnons originates from the non-co-planner spin configuration.

- Color Coding,
  - Red line $\rightarrow$ Upper band touching
  - Blue line $\rightarrow$ Middle band touching
  - Black line $\rightarrow$ Lower band touching

- Chern Number changed by $\pm 2$, if, band touches at any point Gamma-point

- Chern number changed by $\pm 1$, if, band touches at Gamma-point.

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Thermal Hall Conductivity and it’s derivative

• The Thermal Hall conductivity is given by,

\[ \kappa'_{xy} = \frac{\kappa_{xy}}{k_B} = \frac{T'}{(2\pi)^2} \sum_n \int_{BZ} c_2(\rho_n, \mathbf{k}) \Omega_{xy}^n(\mathbf{k}) d^2k, \]

\[ \rho_s^T(\mathbf{k}) = [\exp(E_s^T(\mathbf{k})/T) - 1]^{-1} \]

\[ c_2(\rho) = (1 + \rho)(\log \frac{1+\rho}{\rho})^2 - (\log \rho)^2 - 2 \text{Li}_2(-\rho) \]

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Thermal Hall Conductivity and its derivative

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Thermal Hall Conductivity and it’s derivative

• At topological band transition point the derivative of conductivity is divergent.

• The divergence is logarithmic in nature for Dirac point. (Phys. Rev. B 100, 064412)

• We also derived that the divergence is logarithmic in nature for tilted Dirac and semi-Dirac points.

• The logarithmic divergence is universal in nature. (Phys. Rev. B 101, 214403)
Thermal Hall Conductivity and it’s derivative

- Temperature dependence of logarithmic divergence,

\[
\frac{\partial \kappa'_{xy}}{\partial \rho} = A \left[ \ln \left( \frac{1 + \rho_0}{\rho_0} \right) \right]^2 \exp \left( \frac{\epsilon_0}{T} \right) \rho_0^2,
\]

\[
\rho_0 = \frac{1}{\left( \exp \left( \frac{\epsilon_0}{T} \right) - 1 \right)}
\]

- Two parameters two fit the curve, \( A \) and \( \epsilon_0 \)

- \( A \) depends on band dispersion and Berry curvature. \( \epsilon_0 \) is the band touching point during phase transition.

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Conclusion

• We describe the way of realization of the non-coplanar spin-structure in a realistic Shastry-Sutherland material.
• Perpendicular DM-interaction results in Flux state.
• In plane DM interaction produces Canted Flux state.
• The Magnetic field opens up the band gap and induces non-trivial topological band.
• A zoo of topological phases is found because the Berry phase of magnons originates from non-coplanar spin configuration, and DM-interaction.
• Logarithmic divergence in derivative in thermal Hall conductance is independent of type of Band touching point.
• We established the derivative of thermal Hall conductance as a function of temperature.
Thanks